# Exercises 2: Laplace transform (FYSS 585)

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1. What is the Laplace transform of

$$f(t) = 0 \qquad (t < 0)$$
$$f(t) = \sin(\omega t + \theta) \qquad (t \geq 0)$$

Where $\theta$ is a constant?

2. The Laplace transform *G(s)* of a time function *f(t)* is given by

$$G(s) = \int_0^\infty f(t) \cdot e^{-st} \cdot dt$$

2.1) Find the Laplace transform of *f(at),* where *a* is a positive constant.
Hint: this effect on *G(s)* caused by changing the scale in the domain by a factor *a*.

2.2) If the time function f(t) is multiplied by the exponential term $e^{-at}$. What does the Laplace transform?
Hint: $G_2 = \int_0^\infty \{e^{-at} \cdot f(t)\} \cdot e^{-st} \cdot dt$ represents a complex frequency-shift.

2.3) Show the Laplace transform of the first derivative of *f(t).*

$$G'(s) = \int_0^\infty \frac{d}{dt}\{f(t)\} \cdot e^{-st} \cdot dt$$
$$G'(s) = s \cdot G(s) - f(0)$$

Where *f(0)* is the initial value of *f(t)* evaluated at *t=0*

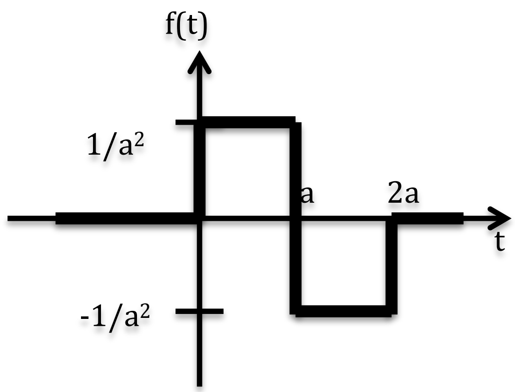
2.4) Find the Laplace transform of the integration of *f(t)* between the limits *t=0* to *t=T*

$$G_4(s) = \int_0^\infty \left\{\int_o^T f(t) \cdot dt\right\} \cdot e^{-st} \cdot dt$$

3. Find the Laplace transform *F(s)* of the function *f(t)* shown in figure below. Also find the limiting value of *F(s)* as *a* approaches zero.

   Hint: The function *f(t)* can be written

$$f(t) = \frac{1}{a^2}1(t) - \frac{2}{a^2}1(t-a) + \frac{1}{a^2}1(t-2a)$$




4. Find the inverse Laplace transform of *F(s),* where

$$F(s) = \frac{1}{s(s^2+2s+2)}$$

   Draw any poles and zero on an s-plane plot.

   Hint : $s^2 + 2s + s = (s+1+j1)(s+1-j1)$


Reference:

K. Ogata, *Modern Control Engineering,* Englewood Cliffs, N.J.: Prentice Hall, Inc., 1990.
P. A. Lynn, An introduction to the analysis and processing of signal, N.J: John Wiley & Sons, Inc., 1973.