Lab 4 - Ziegler-Nichols rules for tuning PID controllers.

Harry J. Whitlow
Department of Physics
University of Jyväskylä
Finland

1 Introduction

An essential instrument in many industrial control situations is the use of a PID (proportional-integral-differential) controller to control some plant. Examples are the stabiliser on passenger ships and temperature control of ovens for oxidation and diffusion for processing semiconductors. The situation is illustrated in Fig. 1. The transfer function for the PID controller

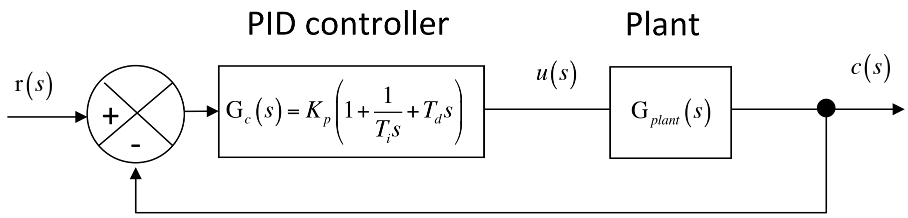

Figure 1: PID control of plant

is:

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \quad (1)$$

The transfer function $G_{plant}(s)$ is generally mathematically unknown and often has to be treated as a black box. It is then necessary to tune the parameters, K_p, T_i and T_d in equation 1. This is known as control tuning. K_p, T_i and T_d are the proportional gain, integration time and differentiation time constants, respectively. The problem is we have three unknowns and how do we find optimum values?

Ziegler and Nichols suggested some rules for tuning the PID parameters, K_p, T_i and T_d based on (i) measuring the transient response of the plant, or (ii) measuring the onset of sustained oscillations as the proportional gain, K_p is increased.

I*n this laboratory work some general guidelines and helpful LabVIEW VIs are provided to explore PID controllers and Ziegler-Nichols tuning. Not all details are given and you are expected consider this as an open-ended lab. Certainly about 1-2 days is needed to work through everything.*

2 The Plant - a simple oven

For our plant we take a simple oven based on a resistor that acts as a heater element. This is in contact with a thermistor that measures the temperature. An example of the use of such small simple heater plant is in the microfluidic replication of DNA using the polymerase chain reaction. This is based on thermal cycling the sample at (i) 94-96 °C to convert double strand DNA to single strand.(ii) Annealing at 55-65 °C to bond the primers to form DNA-DNA by hydrogen bond formation. (iii) Heating at 75-80 °C to synthesise a new DNA strand by polymerase enzyme. (iv) Final hold of the reaction at 4-15 °C. Extremely good temperature control is needed to ensure optimal functioning of the enzyme.

In our case it is intended the oven is powered from the +5 V supply on the NI USB-6009 interface. This must not exceed a current drain of 200 mA. The resistor is 39 Ω which allowing for the 1.2 V V_{ce} voltage drop across the switching transistor draws about 100 mA and generates 0.4 W. This limits the temperature to about 40-45 °C in our test model.

The heater is operated in Pulse Width Modulated (PWM) mode in which the on/off time is modulated proportional to the power level from 0 to 100%. Fig. 2 shows the circuit. This is done by turning the heater on and off for times proportional to the desired power.

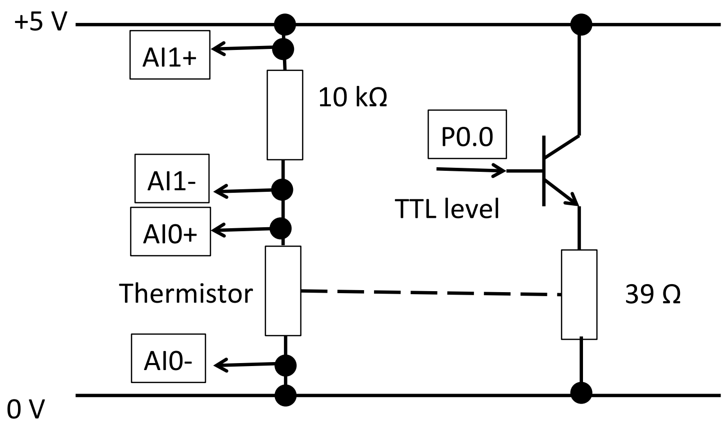

Figure 2: Micro temperature-controlled oven circuit. The heater is the emitter follower that is turned on and off by writing a TTL level to the base of the npn- transistor. The terminal designation on the NI USB-6009 is denoted by the rectangular boxes.

The temperature is measured by a 4-terminal measurements of the voltage across the thermistor and a 10 kΩ shunt resistors. A VI: *V2temp.vi* calculates the temperature in units of °C from the two measured voltages.

Note that is may be necessary to change the DAQmx vis to take account of the different LabVIEW environments in windows and Mac OSX. The ini-

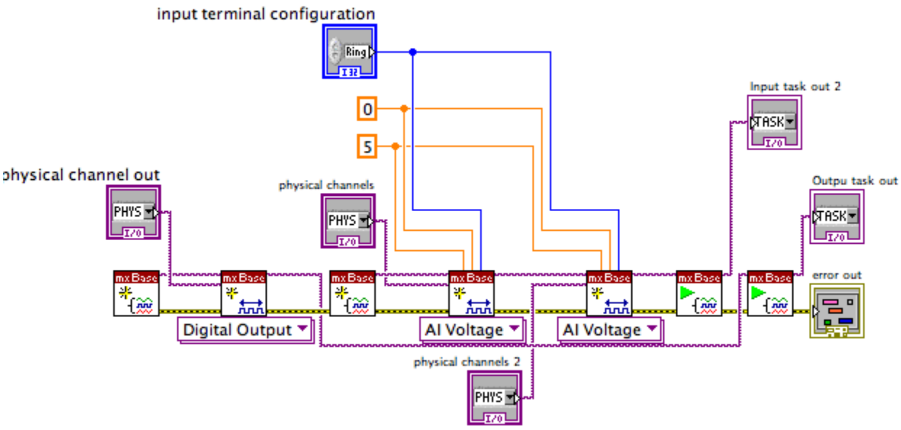

Figure 3: Initialisation VI for the oven (The plant) *Plant-init.vi*.

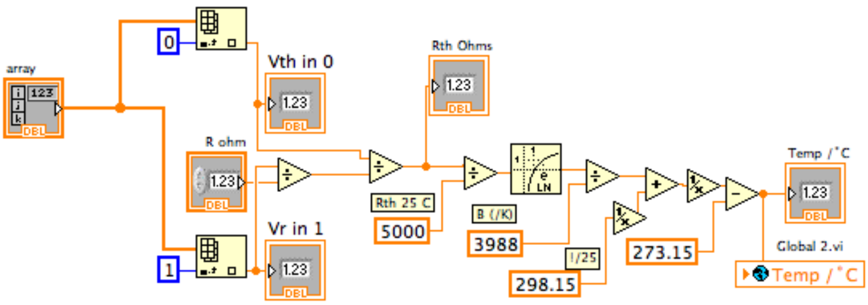

Figure 4: Conversion VI that translates voltages from the thermistor to temperature.

tialisation is carried out by the VI: *Plant-init.vi*. This does the initialisation for the VI called *Plant.vi*. This is shown below in Fig 3. One more sub-VI is important. This converts the voltages from the thermistor measurement to temperature in °C (Fig. 4). This uses the relation:

$$T\,(K) = \frac{1}{\dfrac{1}{T_0} + \dfrac{1}{B} Log_e\left(\dfrac{R}{R_0}\right)}, \tag{2}$$

where, R_0 is 5 kΩ at 298.15 K (25 °C) and B is 3988 K The VI: Plant.vi (Fig. 5) first reads the thermistor and obtains the temperature and then outputs the PWM pulse to the heater to heat the sample. The Plant VI input $u(t)$ a power level $0 \longrightarrow 1$ and returns the temperature of the heater $c(t)$. Fig 1. The transfer function is: $G_{plant}(s)$. Fig. 1.

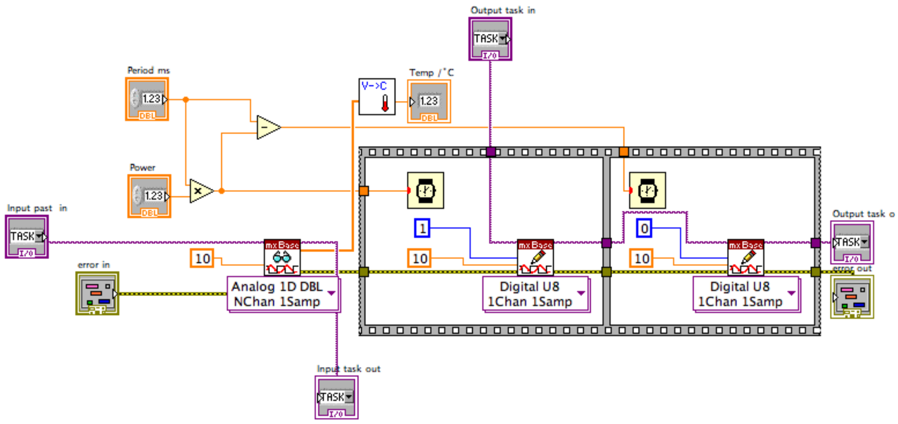

Figure 5: Block diagram of the *plant.vi*.

3 First Ziegler and Nichols rule

Our simple micro-oven has an approximately s-shaped unit-step response curve. This is characteristic of a first-order system with transport lag. A wide range of systems have such a response such as a chemical reactions RC circuits and tank filling devices. This has a transfer function of the form:

$$\frac{C(s)}{U(s)} = \frac{Ke^{-Ls}}{Ts+1}. \tag{3}$$

In equation 3 L is the onset-delay and T the time difference between the interception of a straight line to the s-shaped curve at the inflection point with the $C(s) = 0$ and $C(s) = K$ lines. See Fig. 6. The first Ziegler-Nichols rule is based on measuring the transient response for the plant to determine L and T, (Fig. 6).

3.1 Plant-impulse VI

The VI: *Plant4.vi* demonstrates how the plant.vi is used. It can be used, or better still adapted, to measure L and T for our micro-oven. The VI is shown in Fig. 9. When the switch is thrown the power level $u()$ is applied to the input. The data is mapped as a graph in terms of the number of passes through the loop. Fig. 10 shows the front panel.

Task 1: *You should determine L and T for the micro-oven using LabVIEW.*

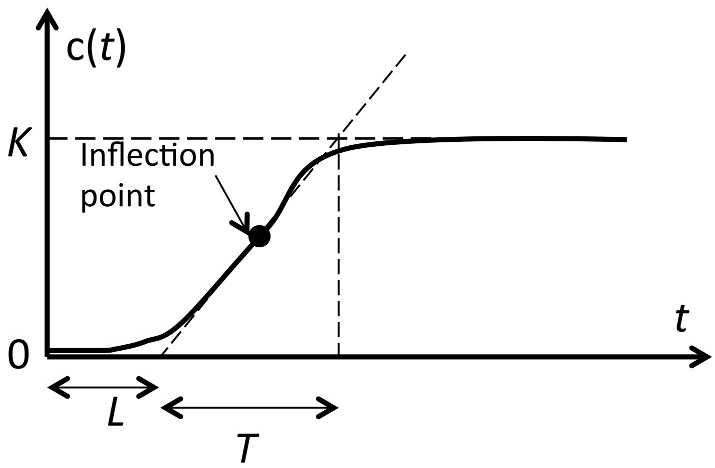

Figure 6: Step-response curve for Equation 3 showing the parameters for the First Ziegler-Nichols rule.

4 Step-response response function rule

Table 1. Step-response based Tuning rule of Ziegler-Nichols

Controller mode	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\left(\frac{T}{L}\right)$	$\frac{L}{0.3}$	0
PID	$1.2\left(\frac{T}{L}\right)$	$2L$	0.5L

Then for a PID controller the response according to the step-response tuning rule is:

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \tag{4}$$

$$= 1.2\left(\frac{T}{L}\right)\left(1 + \frac{1}{2Ls} + \frac{Ls}{2}\right) \tag{5}$$

$$= 0.6\frac{\left(s + \frac{1}{L}\right)^2}{s} \tag{6}$$

This has a pair of zeros at $s = -1/L$ and a pole at the origin.

Task 2: Use Matlab to investigate the effect of K_p, T_i and T_d are the proportional gain, integration time and differentiation time constants, respectively on the step response of the composite controller-plant transfer

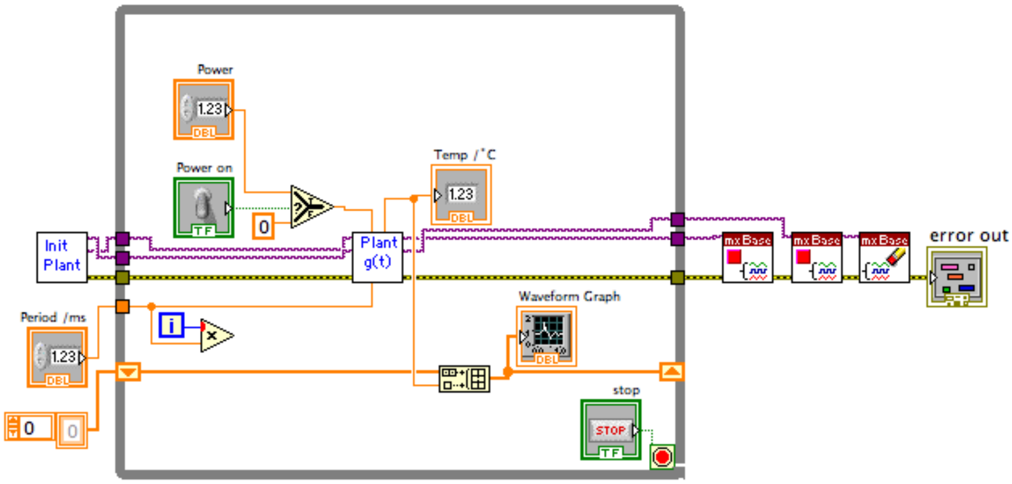

Figure 7: Step-impulse response curve test circuit for *Plant.vi*.

function (See Fig. 1). Hint: expand the exponential in the numerator of Equation 3 as a polynomial expansion and take the first terms. What is the implication of this? Use the values of T and L for the micro-oven determined in Task 1. How well do the tuning values selected using Ziegler-Nichols tuning (Table 1) function?

5 LabVIEW implimentation of PID control

$$G_c(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s\right). \tag{7}$$

Can be transformed to the time domain to give :

$$u(t) = K_p \left[e(t) + \frac{1}{T_i}\int_0^t e(t)\,\mathrm{d}t. + T_d \frac{\mathrm{d}e(t)}{\mathrm{d}t}\right]. \tag{8}$$

Where, $e(t) = r(t) - c(t)$, as usual for a closed loop (Fig. 1). Fig. 9 presents the block diagram for the LabView implementation of the PID controller. By appropriate setting of $T_i \longrightarrow \infty$ and $T_d = 0$ the controller functions as a P-controller. If only $T_d = 0$ then it functions as a PI-controller. Fig. 10 shows the front panel of the controller showing controlling action.

Task 3: Set-up the PID controller in LabVIEW. Start with P-, then PI- and PID control respectively. Investigate the effect of changing K_p, T_i and T_d. Note that the power level is restricted between 0 and 1. (This

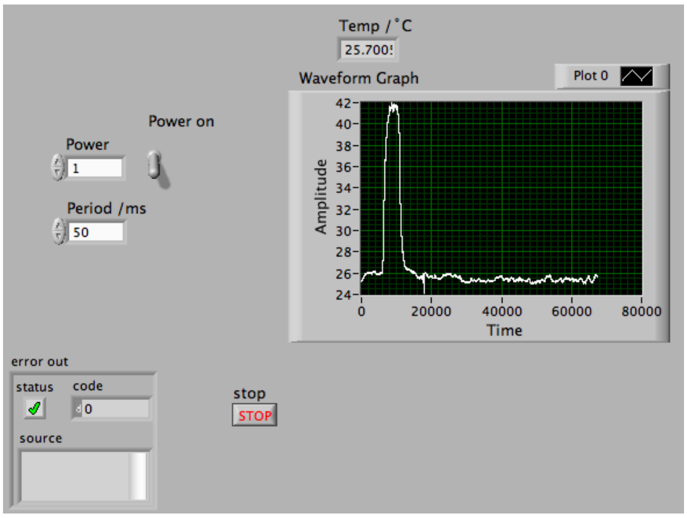

Figure 8: Front panel for the block diagram in Fig. 9.

means if you go outside these limits the controller becomes very non-linear. It does however remain stable.) A transient can be introduced by switching between two set-points. Compare the stability for P, PI and PID modes. A disturbance can be introduced with the compressed gas aerosol jet. It is noteworthy that the temperature in the P-mode always shows a small negative final error value. Consider why this might be?

Task 4: Use the Ziegler-Nichols parameters from Table 1 with the data from Task 1 and see if this gives an acceptable performance. If not you can alter the PID-2.vi to get things more acceptable.

6 Critical gain and period Ziegler-Nichols PID tuning rule

In the second method we set $T_i \longrightarrow \infty$ and $T_d = 0$ so the controller functions as a P-controller only. Then increase K_p to the point where sustainable oscillations just start. Be careful that the power level $u(t)$ oscillates between 0 and 1 so the system is not unlinear. Then at this critical point measure the K_{cr} and P_{cr}.

Table 2. Critical-gain based Tuning rule of Ziegler-Nichols

Controller mode	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\left(\frac{P_{cr}}{1.2}\right)$	0
PID	$0.45K_{cr}$	$\left(\frac{P_{cr}}{2}\right)$	$\left(\frac{P_{cr}}{8}\right)$

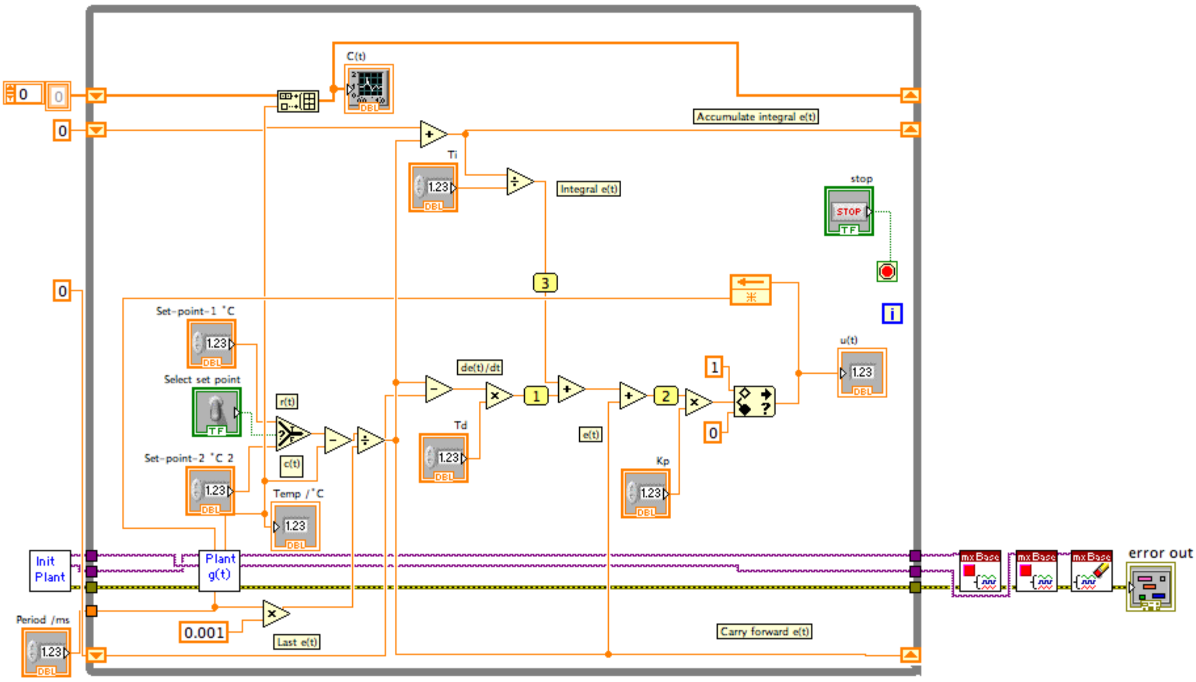

Figure 9: Block diagram of PID controller *PID-2.vi*.

As for the case of the Step-impulse we can determine the position of the poles and zeros by substituting from Table 2:

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \tag{9}$$

$$= 0.6K_{cr}\left[1 + \left(\frac{2}{sP_{cr}}\right) + \left(\frac{sP_{cr}}{8}\right)\right] \tag{10}$$

$$= 0.075K_{cr}P_{cr}\left[\frac{\left(s + \dfrac{4}{P_{cr}}\right)^2}{s}\right] \tag{11}$$

Equation 6 has a pair of zeros at $s = -4/P_{cr}$ and a pole at the origin.

Task 5: K_{cr} and P_{cr} Measure K_{cr} and P_{cr} for the P-controller. You can take $T_i = 10^{12}$ s. This is essentially infinity. You might want to modify the block diagram to measure the period accurately. (e.g. set up a data-logger and fit using or Matlab (Fourier Analysis?) Use the values of K_p, T_i and T_d in the PID and see how well it responds to disturbances and transients.

Task 6: Often one wants to modulate temperature. Using the optimal values of K_p, T_i and T_d modify the block diagram to follow a sawtooth

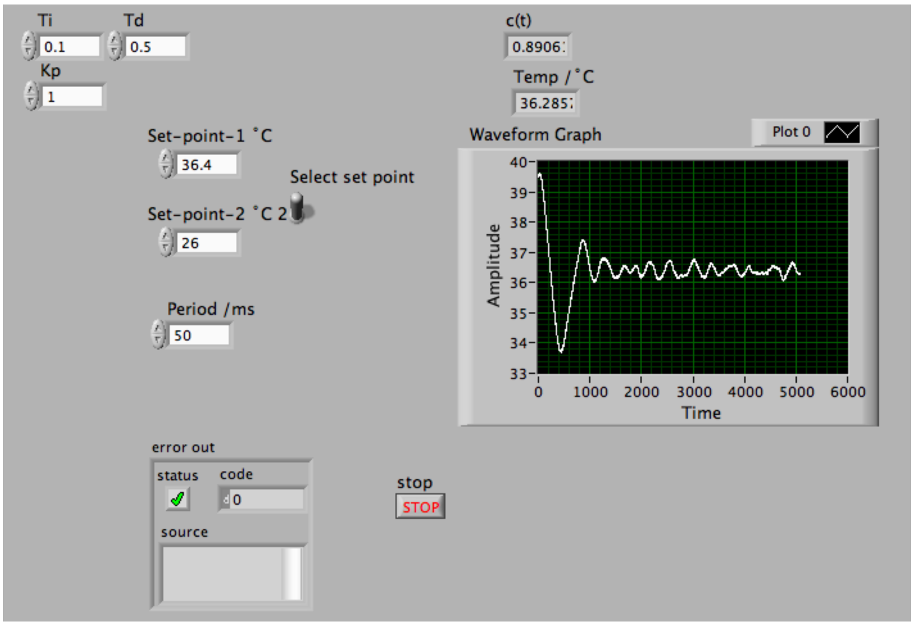

Figure 10: Front panel of PID controller *PID-2.vi*. The controller is operational and it can be seen controlling. (The values of K_p, T_i and T_d are not representative.)

ramped temperature profile and a square wave temperature profile. What happens when the frequency of these waveforms is increased?

Task 7: Consider the position of the poles and zeros in $G_c\,()$. what does the Bode plot look like. (No need to do this in Matlab - a sketch will do.

Task 8: Many systems are not ideal and the plant dynamics are not ideal. (An example is our simple micro-oven.) The plant dynamics cannot be expressed analytically. (An example is the Pelletron Accelerator where the Terminal Voltage Stabiliser is a PI controller. Often, and provided there are no integrating terms in the plant the Ziegler-Nichols rule give a good approximation that can be used as a starting point. Consider when it is appropriate to measure the transient response and when the the critical gain approach is better. Comment briefly in 1-2 sentences your thoughts on this.

Useful References

1. K. Ogata, *Modern control engineering*, 3rd Ed. (Prentice-Hall,1997).
2. G.F. Franklin, J. D. Powell, A. Emani-Naeini, *Feedback control of dynamic systems*, Addison-Wesley,3rd Ed. (Reading MA, 1994).