## Lab 5 FYSS 585

# Spatial domain and Fourier optics analysis of images

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### Objective

The goal of this lab is to investigate image analysis techniques in the spatial and frequency domains.

### Introduction

One of the most significant developments in recent years has been digital imaging. This has all but replaced wet chemical techniques in the darkroom. In this exercise we investigate some of the techniques based on numerical analysis and processing of digital images. The laboratory exercise can be done using Matlab, or the image processing program – ImageJ that is available as freeware from the National Institute of Health. http://rsbweb.nih.gov/ij/ . The use of Matlab or ImageJ allows a more objective handling of scientific images than is possible with more artistically oriented software such as iPhoto, Photoshop and Aperture

Two dimensional image data is essentially digitised two dimensional functions, $f(x,y)$ to produce matrices where each element, $(x_i, y_j)$ represents a pixel. The pixel contents correspond to the intensity. Colour images are generally made up of a number of matrices that correspond at least to the primary colours (R,B,G), but may also contain other channels. Images can in general contain more dimensions than two, for example, $(x_i, y_j, z_k)$ and $(x_i, y_j, z_k, t_l)$ where the former is referred to as a voxel and the latter includes time-steps as well. Often, three dimensional images are made up of a series of 2D images arranged as a "stack" in the third dimension. Here we restrict ourselves to two dimensional images, however the techniques are perfectly general and the processes can be extended rather trivially to higher dimensions.

Download the image files A.png and B.png. A is a on-axis scanning transmission ion microscopy image of a human fibroblast cell [H. J Whitlow, R. Norarata, M. Q. Ren, T. Osipowicz, J.A. van Kan, J. Timonen, F.Watt, Microelectronic Eng. (In press)]. B is an image of the front door of Bletchly Park House, the British WWII code-breaking centre [H.J. Whitlow].

### Pixel-wise transforms

1. Load: A.png into the program e.g. `A = imread('A.png');` in Mathlab. This can be displayed using: `imshow(A);`. Pixel-wise transformations work on each pixel individually. The simplest is just to multiply the matrix A by two: `C=A*2.;` then plot the images.
```
>> figure
>> subplot(1,2,1),imshow(A);
>> subplot(1,2,2),imshow(C);
```

2. Plot a histogram of the pixel contents in image A and investigate how changing the gamma influences the image and histogram.

3. Make a thresholding transformation that shows pixels that exceed 25% of the maximum in image A as white (255) and all others black (0).

4. Make a transformation that produces a negative image of image A.

### Spatial transformations

As their name suggest spatial transformations are based on the spatial relations between pixels. For example a mean filter replaces a pixel by the average of $n{\times}n$ surrounding pixels. The $n{\times}n$ matrix, where $n$ is odd, that describe how the spatially related pixels to transform to pixels in the transformed image is called the *kernel*. For a 3×3 mean we have a kernel:

$$\begin{bmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix}.$$

5. For the image B take the 3×3 mean. Compare the original and transformed image side by side.

6. The median filter where the kernel function takes the median value of the surrounding pixels is particularly useful for removing "salt and pepper" noise. Perform a 3×3 median filter transformation on image B. Compare the results with point 5. above.

**Frequency domain transformation**

Just as time domain signals can be transformed to the complex frequency domain by a Fourier transform we can also perform a 2-D Fourier transform on images. (n.b. if you use ImageJ by default, it displays a log intensity image of the 2D power spectrum after Fourier transformation. Select complex to get the real and imaginary components. In Matlab there is function to switch quadrants) By convention zero frequencies are at the centre of the image.

7. Take the 2D Fast-Fourier transforms of images A and B. Calculate the power spectrum. Compare the output images of the resulting transforms with the original. Explain what you observe? Why is there relatively more high frequency information in B than A?

8. Take the inverse Fast-Fourier transform of the complex spectra produced in point. 7. Confirm the result is identical to the images A and B. (Some poor FFT routines work in single precision only giving some distortion.)

9. Make a circular bandpass mask (already in ImageJ) with a cut off in frequency below 3 pixels and above 40 pixels radius. Apply this to the Fourier transformed image A. Explain the changes in the image? What is the effect of increasing the lower cut-off radius to 20 pixels? What happens if we increase the higher cut-off frequency to 500 pixels radius? Why?

10. Make a frequency domain filters to detect edges.

11. Calculate the power spectrum of image A. Explain the shape?

12. Calculate the 2D autocorrelation function of image A. (Fourier transform of power spectrum of A.). What features in the autocorrelation function can one attribute to features in the image?